Investigation into the Construction of Parallel Plate Capacitors

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Abstract

We designed and built two capacitors based on the parallel plate model: one consisting of two parallel disks separated by paper; and another consisting of many dielectric and conducting layers. We were able to achieve capacitances of 1.21nF and 61.7nF respectively. Modifying our dielectric by adding "Inductor Oil" further increased each roughly by a factor of 2 to 2.15nF and 121.8nF. Our results confirmed the predicted $C = \epsilon A/d$.

Theory of Parallel Plate Capacitors

Using Gauss's Law, it is easy to derive an expression for the capacitance per unit area for two infinite conducting parallel plates, separated by a dielectric,

$$\frac{C}{A} = \frac{\epsilon}{d} \qquad \text{from} \qquad C = \frac{\epsilon A}{d}$$

where d is the distance between the two plates and ϵ is the dielectric constant of the material. Since it is quite difficult to get *infinite* plates in real life, it is useful to notice what the difference is between a set of infinite plates and a set of finite plates. In the case of two infinite plates, the electric field flux lines are normal to each surface, going from one to the other. In the case of the finite plates, the flux lines near the edges will be curved outwards; all the flux will not pass through the area under the plate. These are called the *fringing fields* and this is the important difference between the infinite case and the finite case.

If we consider a finite portion of the infinite plates, we get the capacitance $C = \frac{\epsilon A}{d}$, where A is the surface area of the portion of the plates in our portion. This expression will be a good approximation for the finite plate capacitor if there is little fringing flux, that is — when both the length and width dimensions of the plates are much greater than the separation distance, d. Our TA Ben was nice enough to graph the percent error vs. the l/d ratio for a two–dimensional case, where l is the length of a plate. It indicates that if $l = 10d$, the percent error will be 50%, and if $l = 100d$, the percent error will be only 10%. When $l > 10^3 d$, the error is virtually zero.

First Design

There were several issues constraining our first design. First, we wanted to minimize the fringing fields; second, we obviously wanted to be able to construct it without too much trouble; and finally, we wanted to maximize the capacitance.

In order to minimize the fringing fields, several conditions must be met. The first is that the separation distance must be small relative to a length dimension of the plate. This is easy to satisfy by using a very thin dielectric layer and a plate on the order of several centimeters long. The second condition is that the amount of edge length must be minimized for the given area. This was accomplished by using circular plates.

The last two issues deal with charge distribution on the plates. In the ideal infinite plate case, the charge distribution is uniform. Charges tend to be more concentrated on sharp edges of conductors, so in order to resemble the infinite plate case by reducing fringing fields, we want to reduce the "sharpness" of the edges. On the other hand, since electric flux lines are always normal to a conducting surface, we wanted to minimize the edge surface area so as to minimize the fringing fields. It is hard to satisfy both of these last two conditions at once, because a thin plate tends to have sharper edges.

After searching through tables of dielectric constants of materials that we could use, we decided that since all the possible safe dielectrics we could get our hands on had ϵ_r in the 1 to 4 range, it would probably be more effective to choose a thin material over a thicker one with a higher ϵ_r. This is confirmed by $C = \epsilon A/d$. We planned to experiment with common substances such as paper, high voltage paper from Plasma Physics, plastic, and Reynolds Wrap.

Construction of First Design

After discussion of these issues, we decided to use two round plates of copper, with a layer of some thin dielectric between them. We would then put a phenolic plate on either side of the copper plates and compress the assembly with C–clamps. Finally, we decided to solder two leads coming off the disks on the edge to connect to the rest of the circuit.

We in part chose these materials because Joel and Stefan work at Plasma Physics in Chamberlin Hall, and had access to scrap material and tools there.

The dimensions for our plates were determined by the size of circular drill bit we had access to. Our copper plates had radius 4.8cm. We tried to round off the edges of the plates with a file to reduce fringing fields and make them less likely to cut someone.

Measurement

Originally, we proposed a method of measuring the capacitance by using our capacitor in a RC circuit. We would put our capacitor in series with a resistor and voltage source, and then measure the time it takes for the voltage across the capacitor to rise to 63% of the power supply voltage. Then it is easy to calculate $C = \tau / R$. This equipment is available in the ECE270 Laboratory.

The Plasma Physics shops in Chamberlin had, among lots of other interesting hardware, a Capacitance Meter. We checked the values it reported against some labeled capacitors to make sure it was accurate and then used this for our subsequent measurements.

Results of First Design

First, we tried regular copier paper as the dielectric. With a micrometer from the lab, we measured the thickness of the paper we were using to be 0.08mm when compressed. We sandwiched the paper between the two copper plates, surrounded those by the phenolic blocks, and then tightened the C–clamp down on the assembly. The capacitance meter reported 1.21nF.

The CRC handbook reported the ϵ_r for paper to be somewhere between 2 and 4. For the sake of calculations, let's use 2, which yields a theoretical capacitance

$$C = \frac{\epsilon_o \epsilon_r A}{d} = \frac{(8.854\text{x}10^{-12})(2)(\pi 0.048^2)}{0.08\text{x}10^{-3}} = 1.6\text{nF}$$

Our actual and theoretical values differ by 27%, but that isn't really very helpful because we had no way to measure the ϵ_r of paper, besides knowing that it was probably between 2 and 4.

We also tried other other dielectrics, but found that either they were too thin and caused a short when compressed (tissue and Reynolds Wrap), or that they were too thick and yielded capacitances much less than 1nF (high voltage paper and plastic bag material).

Second Design

We weren't satisfied with a measely 1.21nF, we wanted more!

We had already searched for and tried all the possible sheet–type dielectrics that we could find in the Plasma Lab in our original model, so we decided to increase the capacitance by using several layers of larger plates, separated by dielectric layers. Each alternate conducting layer goes to a different lead. So "even" layers go to one lead and "odd" layers go to the other. This whole unit can be viewed as 7 capacitors in parallel.

We also thought that maybe we could increase the capacitance by somehow increasing ϵ_r of the dielectric. There was an old can of "Inductor Oil" which we decided to try soaking the paper in to test its effects. We found later this oil is called "Diallyl."

Construction of Second Design

We decided to use 20x26cm plates, because that was the largest we could make our surrounding phenolic blocks from the material we had. It would have been a little better in terms of fringing fields to have round plates, but we didn't want to reduce the size of our plates at all. The reduction of area would have reduced the capacitance more than the losses due to the fringing fields using rectangular plates.

We decided to use 8 layers of aluminum foil, which was the only conducting material we could get enough of. We also chose to use regular copier paper as in the first design. Our design was 8 layers of aluminum foil, interspersed with 7 layers of paper. We had to use very small pieces of Scotch Tape to hold the layers in place because the paper and aluminum foil pieces were very slippery. We then put the phenolic blocks on both sides and compressed the unit with 3 C–clamps.

The aluminum foil pieces each had an extra 0.75" square tab on one corner to clip the alligator clips onto.

Results of Second Design

When we tried the whole assembly with no oil, we measured a capacitance of $C = 0.0617\mu\text{F}$. The theoretical capacitance of this, assuming again $\epsilon_r = 2$,

$$C = \frac{n\epsilon_o\epsilon_r A}{d} = \frac{(7)(8.854\text{x}10^{-12})(2)(0.20)(0.26)}{0.08\text{x}10^{-3}} = 80.6\text{nF}$$

Our actual value differs from this by 27% again. Notice that for our first design, the measured C was also 27% low. Does this mean we were "just wrong" twice? I think it means that the ϵ_r for *our* paper was actually lower than the range 2 – 4 given in the CRC handbook. If we, for the moment, assume as given our measured C values, and we re-calculate the ϵ_r of paper from that data, we get

$$\frac{1.6}{2} = \frac{1.21}{\epsilon_{r1}}, \qquad \epsilon_{r1} = 1.51$$

and

$$\frac{80.6}{2} = \frac{61.7}{\epsilon_{r2}}, \qquad \epsilon_{r2} = 1.53$$

Notice that there is very good agreement between these two calculated ϵ_r values for both the small single layered capacitor and the large multilayered capacitor.

Before we wanted to make a mess trying the big capacitor with the oil, we put the small round capacitor together using a piece of oil-soaked paper as the dielectric to see if it would increase the capacitance at all. It did, yielding a measured capacitance of $C = 2.15$nF, almost 2 times the value we got for regular paper.

So we disassembled our large capacitor and soaked each piece of paper in the "Inductor Oil." We then assembled the layers on the phenolic block and clamped it together. This time we didn't have to use tape to hold the layers together because the oil's surface tension stuck them together for us.

The Cap Meter reported "Error — Short." So we disassembled the layers and found that a piece of grit had penetrated 4 layers when we tightened the clamps and had shorted the layers. To fix this, we replaced the punctured dielectric layers and reassembled, more carefully this time.

We slowly tightened the C–clamps while measuring C. When no pressure was applied, $C = 0.0947\mu$F. As we tightened, it finally rose to $C = 0.1218\mu$F after we had applied a large amount of pressure by tightened each C–clamp with a short hollow pipe to get more leverage and torque. This increase in capacitence lends credence to the $1/d$ relationship.

We had no way to measure the ϵ_r for this oil, but if we assume the measured C is correct, we can go back and find ϵ_r for both the small and large capacitors with oil by comparing them to the dry case, using ϵ_r for paper to be the average of 1.51 and 1.53, $\epsilon_{rdry} = 1.52$,

$$\frac{2.15}{1.21} = \frac{\epsilon_{r1oil}}{1.52}, \qquad \epsilon_{r1oil} = 2.7$$

and

$$\frac{121.8}{61.7} = \frac{\epsilon_{r2oil}}{1.52}, \qquad \epsilon_{r2oil} = 3.0$$

These don't correspond quite as nicely as the values for the first part, but there is still only a 10% difference between 2.7 and 3.0.

Conclusions

The increase in C as we tighted the clamps suggests that the $1/d$ relationship is correct, although we didn't design our experiment to test this. If we wanted to conduct such a test, we would measure the capacitence of our little capacitor with different numbers of dielectric layers. In each case, we would use the same pressure to try to keep d constant. Then we could plot C as a function of $1/d$ and the more straight the line, the better our demonstration that $C \propto 1/d$. We would expect the slope of this line to be ϵA.

Our data also supports the relationship that $C \propto A$. If we take the ratio of the capacitnces

$$\frac{A_2}{A_1} = \frac{(7)(0.2)(0.26)}{(\pi)(0.048^2)} = 50.3$$

and compare that to our measured capacitences

$$\frac{C_{2dry}}{C_{1dry}} = \frac{61.7}{1.21} = 51.0 \qquad \text{and} \qquad \frac{C_{2oil}}{C_{1oil}} = \frac{121.8}{2.15} = 56.7$$

The dry capacitors were only 1.39% away from the expected increase in C, and the wet capacitors were 12% away from the expected increase in C. These results confirm the $C \propto A$ relationship.

The significant error we had showed up in the retro–calculations of ϵ_r for the oil dielectric. I think the 12% error for the change in A vs C for the wet capacitor was because of the same reason that gave the 10% difference between the calculated ϵ_r for the wet dielectric. This difference may be attributed to a changing composition of dielectric as we tightened the clamps. The oil dielectric consisted of paper sheets that were soaked in oil. The oil, "Diallyl," is supposed to have an ϵ_r around 4 — 6, and the ϵ_r for paper is less than that. As we tightened the clamps, oil was squeezed out of the paper like water from a sponge. This would decrease ϵ_r for the combination of paper and oil.

We had no way to keep pressure constant between the two capacitors, so this is probably the source of the error.

Going Further

We were able to calculate ϵ_r retroactively because we had a measured C, and known A and d. Once we have a material with a known ϵ_r, our capacitor becomes a "proof of principle" device to measure dielectric constants of materials. An actual device would have to keep track of area and distance, measure the capacitence, and then compute an adjusted ratio of the capacitences to reveal the ϵ_r for the unknown material, as we did by hand for this analysis.